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A component of superconnection of 11-dimensional curved superspace at second order in anticommuting coordinates

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Abstract

We calculate a component of connection superfields and Lorentz superparameter at second order in anticommuting coordinates in terms of the component fields of 11-dimensional on-shell supergravity by using 'Gauge completion'. This configuration of superspace holds the κ-symmetry for supermembrane Lagrangian and represents 11-dimensional on-shell supergravity.

1 Introduction

In recent developments of string theory, the deep connection between supergravity and super Yang-Mills theory has clarified. One remarkable example is the AdS/CFT correspondence [1]. In this researches, many supergravity theories which can be obtained by dimensional reduction from 11-dimensional supergravity have played important roles [2].

On the other hand, some years ago, T. Banks, W. Fischler, S. H. Shenker and L. Susskind (BFSS) proposed that Matrix theory gives a complete description of light-front M-theory [3]. It had been proposed as a theory of D0-branes by E. Witten [4]. The candidate of its extension on curved backgrounds is the supermembrane theory. It is described as nonlinear sigma model [5] and couples to 11-dimensional superspace backgrounds that satisfy a number of constraints which are equivalent to 11-dimensional on-shell supergravity [6].

Thus, it is important that getting much knowledge of 11-dimensional superspace structure. Nevertheless we have little knowledge of it. By E. Cremmer, S. Ferrara, L. Brink and P. Howe the on-shell conditions and components of superfields up to first order in anticommuting coordinates was investigated [6]. By B. de Wit, K. Peeters and J. Plefka the components of 3-form superfield and part of components of vielbein superfields up to second order in anticommuting coordinates was investigated [7]. In the previous paper, the remaining components of vielbein superfields up to second order in anticommuting coordinates was investigated [8].

In this paper, we compute part of components of connection superfields and the components of Lorentz superparameter at second order in anticommuting coordinates in terms of the component fields of 11-dimensional on-shell supergravity by using 'Gauge completion'. This configuration of superspace holds the κ-symmetry for supermembrane Lagrangian and represents 11-dimensional on-shell supergravity.

The paper is organyized as follows. In section 2, we explain 'gauge completion'. In section 3, we compute parts of the superfields. Our conventions are summarized in Appendix.

2 Gauge Completion

'Gauge completion' was introduced to identify superspace representation as on-shell supergravity [9]. In this section we review this method.

'Gauge completion' is searching for structures of the superfields and superparameters which are compatible with ordinary supergravity. That is to say, supertransformations

(2.5) - (2.7) are identified as transformations in 11-dimensional spacetime (2.1) and the $\theta = 0$ components of superfields and superparameters are identified as the fields and parameters of ordinary supergravity.

2.1 Supersymmetry algebra

Supersymmetry transformations in components formalism are as follows,

$$
\begin{aligned}
\delta_s e_m{}^a &= 2\bar{\epsilon}\Gamma^a\psi_m, \\
\delta_s \psi_m &= D_m(\hat{\omega})\epsilon + T_m{}^{rstu}\epsilon\hat{F}_{rstu} \equiv \hat{D}_m(\hat{\omega})\epsilon, \\
\delta_s C_{klm} &= -6\bar{\epsilon}\Gamma_{[kl}\psi_{m]}, \qquad (2.1) \\
\text{with}, T_m{}^{rstu} &\equiv \frac{1}{288}(\Gamma_m{}^{rstu} - 8\delta_m^{[r}\Gamma^{stu]}), \qquad (2.2)
\end{aligned}
$$

where $\hat{F}(= F_{klmn} + 12\bar{\psi}_{[k}\Gamma_{lm}\psi_{n]})$ is the supercovariant field strength, and $\hat{\omega}(= \omega_m{}^a{}_b + \frac{1}{2}\bar{\psi}_n\Gamma_m{}^a{}_b{}^{np}\psi_p)$ is the supercovariant spin connection. And other notation is the same as that in [8].

Its algebra is as follows,

$$
[\delta_{susy}(\epsilon_1), \delta_{susy}(\epsilon_2)] = \delta_g(\xi_3) + \delta_s(\epsilon_3) + \delta_l(\lambda_3) + \delta_c(\xi_{3mn}), \qquad (2.3)
$$

where

$$
\begin{aligned}
\xi_3^m &= \bar{\epsilon}_2\Gamma^m\epsilon_1 - (1 \leftrightarrow 2), \\
\epsilon_3 &= -\bar{\epsilon}_2\Gamma^n\epsilon_1\psi_n - (1 \leftrightarrow 2), \\
\lambda_3{}^a{}_b &= -\bar{\epsilon}_2\Gamma^n\epsilon_1\hat{\omega}_n{}^a{}_b + \frac{1}{144}\bar{\epsilon}_2(\Gamma^a{}_b{}^{rstu}\hat{F}_{rstu} + 24\Gamma_{rs}\hat{F}^a{}_b{}^{rs})\epsilon_1 - (1 \leftrightarrow 2), \\
\xi_{3mn} &= -\bar{\epsilon}_2\Gamma^k\epsilon_1 C_{kmn} - \bar{\epsilon}_2\Gamma_{mn}\epsilon_1 - (1 \leftrightarrow 2). \qquad (2.4)
\end{aligned}
$$

On the other hand, transformations in superspace formalism are as follows. The supertransformation is equal to

$$
\delta_T X_{M_p \ldots M_1} = \Xi^K \partial_K X_{M_p \ldots M_1} + p\partial_{[M_p}\Xi^K X_{|K|M_{p-1}\ldots M_1]} \qquad (2.5)
$$

for p-form's components. The local Lorentz transformations are equal to

$$
\begin{aligned}
\delta_L E^A &= E^B \Lambda_B{}^A, \\
\delta_L \Omega_B{}^A &= -\Lambda_B{}^C\Omega_C{}^A + \Omega_B{}^C\Lambda_C{}^A - d\Lambda_B{}^A. \qquad (2.6)
\end{aligned}
$$

The supergauge transformations are equal to

$$\delta_G B_{LMN} = 3\partial_{[L}\Xi_{MN]}. \tag{2.7}$$

We obtain the full algebra of these transformations as follows

$$\begin{aligned}[]
[\delta_T(\Xi_1) + \delta_L(\Lambda_1) + \delta_G(\Xi_{1MN}), \delta_T(\Xi_2) + \delta_L(\Lambda_2) + \delta_G(\Xi_{2MN})] \\
= \delta_T(\Xi_3) + \delta_L(\Lambda_3) + \delta_G(\Xi_{3MN}),
\end{aligned} \tag{2.8}$$

where,

$$\begin{aligned}
\Xi_3^K &= \Xi_2^L \partial_L \Xi_1^K + \delta_1 \Xi_2^K - (1 \leftrightarrow 2), \\
\Lambda_{3A}{}^B &= -\Xi_1^K \partial_K \Lambda_{2A}{}^B + \delta_1 \Lambda_{2A}{}^B + \Lambda_{1A}{}^C \Lambda_{2C}{}^B - (1 \leftrightarrow 2), \\
\Xi_{3MN} &= \delta_1 \Xi_{2MN} - \Xi_1^K \partial_K \Xi_{2MN} - 2\partial_{[M} \Xi_{2N]K} \Xi_1^K - (1 \leftrightarrow 2).
\end{aligned} \tag{2.9}$$

2.2 Gauge completion

Firstly, we choose the input data as follows

$$\begin{aligned}
E_m{}^{a(0)} &= e_m{}^a, \\
E_m{}^{\alpha(0)} &= \psi_m{}^\alpha, \\
\Omega_{mb}{}^{a(0)} &= -\hat{\omega}_m{}^a{}_b, \\
\Xi^{m(0)} &= \xi^m, \\
\Xi^{\mu(0)} &= \epsilon^\mu, \\
\Xi_{mn}^{(0)} &= \xi_{mn}, \\
B_{mnl}^{(0)} &= C_{mnl}.
\end{aligned} \tag{2.10}$$

From (2.6), we obtain

$$\Lambda_b{}^{a\ (0)} = \lambda^a{}_b. \tag{2.11}$$

Moreover we introduce the assumption that superparameters do not include the derivative of ϵ. Then, the higher order components in anticommuting coordinates can be obtained by requiring consistency between the algebra of superspace supergravity and that of ordinary supergravity.

If we can represent $\Xi_{MN} = 2\partial_{[M}\Phi_{N]}$, we can choose the gauge as $\Xi_{MN} = 0$ because this superparameters do not change the 3-form superfields (2.7) and the algebra (2.9). Thus we can choose the gauge as follows,

$$\Xi^{(0)}_{\mu N} = 0. \tag{2.12}$$

To obtain the higher order components of superparameters which depend on ϵ, we must calculate the commutation of two supersymmetry transformation.

According to (2.4),(2.9) and (2.17),

$$\begin{aligned}
[\delta_{s1}, \delta_{s2}] E_m{}^{a(0)} &= (\Xi_3^K \partial_K E_m{}^a + \partial_m \Xi_3^K E_K{}^a + E_m{}^b \Lambda_{3b}{}^a)|_{\theta=0} \\
&= (\delta_g(2\bar{\epsilon}_2\Gamma^m\epsilon_1) + \delta_s(-2\bar{\epsilon}_2\Gamma^n\epsilon_1\psi_n) + \delta_c(-2\bar{\epsilon}_2\Gamma^k\epsilon_1 C_{kmn} - 2\bar{\epsilon}_2\Gamma_{mn}\epsilon_1) \\
&\quad + \delta_l(-2\bar{\epsilon}_2\Gamma^n\epsilon_1\hat{\omega}_n{}^a{}_b + \frac{1}{72}\bar{\epsilon}_2(\Gamma^a{}_b{}^{rstu}\hat{F}_{rstu} + 24\Gamma_{rs}\hat{F}^a{}_b{}^{rs})\epsilon_1)) e_m{}^a.
\end{aligned} \tag{2.13}$$

Thus one obtains

$$\Xi^{k(1)}(susy) = \bar{\theta}\Gamma^k\epsilon. \tag{2.14}$$

In the same way, to obtain the higher order components of superparameters which depend on λ we must calculate the commutation of supersymmetry transformation and Lorentz transformation. To obtain the higher order components of superparameters which depend on ξ_{mn} we must calculate the commutation of supersymmetry transformation and gauge transformation. To obtain the higher order components of superparameters which depend on ξ^m we must calculate the commutation of supersymmetry transformation and general coordinate transformation. According to superspace algebra,

$$\begin{aligned}
\delta_{susy} E_m{}^a|_{\theta=0} &= (\Xi^K(susy)\partial_K E_m{}^a + \partial_m \Xi^K(susy) E_K{}^a + E_m{}^b \Lambda_b{}^a(susy))|_{\theta=0} \\
&= \epsilon^\nu \partial_\nu (E_m{}^{a(1)}) + \partial_m \epsilon^\nu E_\nu{}^{a(0)},
\end{aligned} \tag{2.15}$$

while in ordinary supergravity

$$\delta_{susy} e_m{}^a = 2\bar{\epsilon}\Gamma^a\psi_m. \tag{2.16}$$

Thus, one obtains

$$\begin{aligned}
E_\nu{}^{a\ (0)} &= 0, \\
E_m{}^{a(1)} &= 2\bar{\theta}\Gamma^a\psi_m.
\end{aligned} \tag{2.17}$$

By this procedure, the following results had been known [6], [7], [8] .

$$
\begin{aligned}
\Xi^m &= \xi^m + \bar\theta\Gamma^m\epsilon - \bar\theta\Gamma^n\epsilon\bar\theta\Gamma^m\psi_n + \mathcal{O}(\theta^3), && (2.18)\\
\Xi^\mu &= \epsilon^\mu - \frac{1}{4}\lambda_{cd}(\Gamma^{cd}\theta)^\mu - \bar\theta\Gamma^n\epsilon\psi_n{}^\mu + \bar\theta\Gamma^n\epsilon\bar\theta\Gamma^k\psi_n\psi_k{}^\mu + \frac{1}{4}\bar\theta\Gamma^n\epsilon\hat\omega_{nab}(\Gamma^{ab}\theta)^\mu \\
&\quad -\frac{1}{3}\bar\theta\Gamma^k\epsilon(T_k{}^{abcd}\theta)^\mu\hat F_{abcd} - \frac{1}{864}\bar\theta(\Gamma_{ab}{}^{cdef}\hat F_{cdef} + 24\Gamma^{cd}\hat F_{abcd})\epsilon(\Gamma^{ab}\theta)^\mu \\
&\quad +\mathcal{O}(\theta^3), && (2.19)\\
\Lambda_b{}^a &= \lambda^a{}_b - \bar\theta\Gamma^n\epsilon\hat\omega_n{}^a{}_b + \frac{1}{144}\bar\theta(\Gamma^a{}_b{}^{rstu}\hat F_{rstu} + 24\Gamma_{rs}\hat F^a{}_b{}^{rs})\epsilon + \mathcal{O}(\theta^2), && (2.20)\\
\Xi_{mn} &= \xi_{mn} - (\bar\theta\Gamma^p\epsilon C_{pmn} + \bar\theta\Gamma_{mn}\epsilon) + \bar\theta\Gamma^k\epsilon\bar\theta\Gamma^l\psi_k C_{lmn} + \bar\theta\Gamma^k\epsilon\bar\theta\Gamma_{mn}\psi_k \\
&\quad +\frac{4}{3}\bar\theta\Gamma^l\epsilon\bar\theta\Gamma_{l[m}\psi_{n]} + \frac{4}{3}\bar\theta\Gamma^l\psi_{[n}\bar\theta\Gamma_{|l|m]}\epsilon + \mathcal{O}(\theta^3), && (2.21)\\
\Xi_{m\mu} &= \frac{1}{6}\bar\theta\Gamma^n\epsilon(\bar\theta\Gamma_{mn})_\mu + \frac{1}{6}(\bar\theta\Gamma^n)_\mu\bar\theta\Gamma_{mn}\epsilon + \mathcal{O}(\theta^3), && (2.22)\\
\Xi_{\mu\nu} &= \mathcal{O}(\theta^3), && (2.23)\\
E_m{}^a &= e_m{}^a + 2\bar\theta\Gamma^a\psi_m - \frac{1}{4}\bar\theta\Gamma^{acd}\theta\hat\omega_{mcd} + \frac{1}{72}\bar\theta\Gamma_m{}^{rst}\theta\hat F_{rst}{}^a \\
&\quad +\frac{1}{288}\bar\theta\Gamma^{rstu}\theta\hat F_{rstu}e_m{}^a - \frac{1}{36}\bar\theta\Gamma^{astu}\theta\hat F_{mstu} + \mathcal{O}(\theta^3), && (2.24)\\
E_m{}^\alpha &= \psi_m{}^\alpha - \frac{1}{4}\hat\omega_{mab}(\Gamma^{ab}\theta)^\alpha + (T_m{}^{rstu}\theta)^\alpha\hat F_{rstu} \\
&\quad +\bar\theta\Gamma^k\psi_m(T_k{}^{abcd}\theta)^\alpha\hat F_{abcd} - \frac{1}{576}\bar\theta(\Gamma_{ab}{}^{cdef}\hat F_{cdef} + 24\Gamma^{cd}\hat F_{abcd})\psi_m(\Gamma^{ab}\theta)^\alpha \\
&\quad -12(T_k^{abcd}\theta)^\alpha\bar\theta\Gamma_{[ab}\hat D_c\psi_{d]} - \frac{1}{4}(\bar\theta\Gamma_a\hat D_m\psi_b - \bar\theta\Gamma_b\hat D_m\psi_a + \bar\theta\Gamma_m\hat D_a\psi_b)(\Gamma^{ab}\theta)^\alpha \\
&\quad +\mathcal{O}(\theta^3), && (2.25)\\
E_\mu{}^a &= -(\Gamma^a\theta)_\mu + \mathcal{O}(\theta^3), && (2.26)\\
E_\mu{}^\alpha &= \delta_\mu{}^\alpha \\
&\quad -\frac{1}{3}(\Gamma^k\theta)_\mu(T_k{}^{abcd}\theta)^\alpha\hat F_{abcd} + \frac{1}{1728}((\Gamma_{ab}{}^{cdef}\hat F_{cdef} + 24\Gamma^{cd}\hat F_{abcd})\theta)_\mu(\Gamma^{ab}\theta)^\alpha \\
&\quad +\mathcal{O}(\theta^3), && (2.27)\\
\Omega_{\mu b}{}^a &= \frac{1}{144}\{(\Gamma^a{}_b{}^{rstu}\theta)_\mu\hat F_{rstu} + 24(\Gamma_{rs}\theta)_\mu\hat F^a{}_b{}^{rs}\} + \mathcal{O}(\theta^2), && (2.28)\\
\Omega_{mab} &= \hat\omega_{mab} + 2\bar\theta\{e^n{}_a e^k{}_b(-\Gamma_k D_{[m}\psi_{n]} + \Gamma_n D_{[m}\psi_{k]} + \Gamma_m D_{[n}\psi_{k]})\}
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{1}{72}\bar{\theta}(\Gamma_{ab}{}^{rstu}\hat{F}_{rstu}+24\Gamma_{rs}\hat{F}_{ab}{}^{rs})\psi_m+\mathcal{O}(\theta^2), && (2.29)\\
B_{mnl} \;=\;& C_{mnl}-6\bar{\theta}\Gamma_{[mn}\psi_{l]}+\frac{3}{4}\hat{\omega}_{[l}{}^{cd}\bar{\theta}\Gamma_{mn]cd}\theta-\frac{3}{2}\hat{\omega}_{[lmn]}\theta^2 \\
& -\frac{1}{96}\bar{\theta}\Gamma_{mnl}{}^{rstu}\theta\hat{F}_{rstu}-\frac{3}{8}\bar{\theta}\Gamma_{[l}{}^{rs}\theta\hat{F}_{|rs|mn]}-12\bar{\theta}\Gamma_a\psi_{[m}\bar{\theta}\Gamma^a{}_n\psi_{l]} \\
& +\mathcal{O}(\theta^3), && (2.30)\\
B_{mn\mu} \;=\;& (\bar{\theta}\Gamma_{mn})_\mu+\frac{8}{3}\bar{\theta}\Gamma^k\psi_{[m}(\bar{\theta}\Gamma_{|k|n]})_\mu+\frac{4}{3}(\bar{\theta}\Gamma^k)_\mu\bar{\theta}\Gamma_{k[m}\psi_{n]}+\mathcal{O}(\theta^3), && (2.31)\\
B_{m\mu\nu} \;=\;& (\bar{\theta}\Gamma_{mn})_{(\mu}(\bar{\theta}\Gamma^n)_{\nu)}+\mathcal{O}(\theta^3), && (2.32)\\
B_{\mu\nu\rho} \;=\;& (\bar{\theta}\Gamma_{mn})_{(\mu}(\bar{\theta}\Gamma^m)_\nu(\bar{\theta}\Gamma^n)_{\rho)}+\mathcal{O}(\theta^3). && (2.33)\\
& && (2.34)
\end{aligned}
$$

Because the flat geometry had been known, we include the θ^3 term in $B_{\mu\nu\rho}$ for completeness.

Up to first order in anticommuting coordinates, the superfield components was investigated by E. Cremmer and S. Ferrara [6]. $\Xi^{k(2)}, E_M{}^{k(2)}, \Xi^{(2)}_{MN}, B^{(2)}_{LMN}$ was investigated by B. de Wit, K. Peeters and J. Plefka [7]. $E_M{}^{\alpha(2)}, \Xi^{\mu(2)}$ was investigated in ref. [8].

3 Computation

$\Lambda_{ab}{}^{(2)}$ is subject to the following equations,

$$
\begin{aligned}
\epsilon_1^\nu\partial_\mu\partial_\nu\Lambda_{2ab}{}^{(2)}-(1\leftrightarrow 2) \;=\;& \bar{\epsilon_2}\Gamma^n\epsilon_1(\Gamma^k\psi_n)_\mu\hat{\omega}_{kba}-(\Gamma^n\epsilon_1)_\mu\bar{\psi_n}\Gamma^k\epsilon_2\hat{\omega}_{kba} \\
& -\frac{1}{144}\bar{\epsilon_2}\Gamma^n\epsilon_1\{(\Gamma_{ba}{}^{cdef}\hat{F}_{cdef}+24\Gamma^{cd}\hat{F}_{bacd})\psi_n\}_\mu \\
& +\frac{1}{144}(\Gamma^n\epsilon_1)_\mu\bar{\psi_n}(\Gamma_{ba}{}^{cdef}\hat{F}_{cdef}+24\Gamma^{cd}\hat{F}_{bacd})\epsilon_2 \\
& +\frac{1}{6}(\Gamma_{ba}{}^{cdef}\epsilon_2)_\mu\bar{\epsilon_1}\Gamma_{cd}\hat{D}_e\psi_f+4(\Gamma^{cd}\epsilon_2)_\mu\bar{\epsilon_1}\Gamma_{[ba}\hat{D}_c\psi_{d]} \\
& +(\Gamma^k\epsilon_2)_\mu 2\bar{\epsilon_1}(-\Gamma_a\hat{D}_{[k}\psi_{b]}+\Gamma_k\hat{D}_{[b}\psi_{a]}+\Gamma_b\hat{D}_{[k}\psi_{a]}) \\
& -(1\leftrightarrow 2). \qquad (3.1)
\end{aligned}
$$

However, if simply we drive the equation,

$$
\epsilon_1^\nu\partial_\mu\partial_\nu\Lambda_{2ab}{}^{(2)} \;=\; (\bar{\epsilon_2}\Gamma^n)_\nu\epsilon_1^\nu(\Gamma^k\psi_n)_\mu\hat{\omega}_{kba}-(\Gamma^n)_{\mu\nu}\epsilon_1^\nu\bar{\psi_n}\Gamma^k\epsilon_2\hat{\omega}_{kba}
$$

$$
\begin{aligned}
&-\frac{1}{144}(\bar{\epsilon_2}\Gamma^n)_\nu\epsilon_1^\nu\{(\Gamma_{ba}{}^{cdef}\hat{F}_{cdef}+24\Gamma^{cd}\hat{F}_{bacd})\psi_n\}_\mu \\
&+\frac{1}{144}(\Gamma^n)_{\mu\nu}\epsilon_1^\nu\bar{\psi_n}(\Gamma_{ba}{}^{cdef}\hat{F}_{cdef}+24\Gamma^{cd}\hat{F}_{bacd})\epsilon_2 \\
&+\frac{1}{6}(\Gamma_{ba}{}^{cdef}\epsilon_2)_\mu\epsilon_1^\nu(\Gamma_{cd}\hat{D}_e\psi_f)_\nu+4(\Gamma^{cd}\epsilon_2)_\mu\epsilon_1^\nu(\Gamma_{[ba}\hat{D}_c\psi_{d]})_\nu \\
&+(\Gamma^k\epsilon_2)_\mu 2\epsilon_1^\nu(-\Gamma_a\hat{D}_{[k}\psi_{b]}+\Gamma_k\hat{D}_{[b}\psi_{a]}+\Gamma_b\hat{D}_{[k}\psi_{a]})_\nu, \qquad (3.2)
\end{aligned}
$$

this equation is inconsistent because μ and ν in the left-hand side of it are antisymmetric but these in the right-hand side of it are not antisymmetric. Thus we must add terms which are symmetric under interchanging the indices 1 and 2 in the right-hand side of this equation.

$$
\begin{aligned}
\epsilon_1^\nu\partial_\mu\partial_\nu\Lambda_{2ab}{}^{(2)} &= \bar{\epsilon_2}\Gamma^n\epsilon_1(\Gamma^k\psi_n)_\mu\hat{\omega}_{kba}-(\Gamma^n\epsilon_1)_\mu\bar{\psi_n}\Gamma^k\epsilon_2\hat{\omega}_{kba} \\
&-\frac{1}{144}\bar{\epsilon_2}\Gamma^n\epsilon_1\{(\Gamma_{ba}{}^{cdef}\hat{F}_{cdef}+24\Gamma^{cd}\hat{F}_{bacd})\psi_n\}_\mu \\
&+\frac{1}{144}(\Gamma^n\epsilon_1)_\mu\bar{\psi_n}(\Gamma_{ba}{}^{cdef}\hat{F}_{cdef}+24\Gamma^{cd}\hat{F}_{bacd})\epsilon_2 \\
&+\frac{1}{6}(\Gamma_{ba}{}^{cdef}\epsilon_2)_\mu\bar{\epsilon_1}\Gamma_{cd}\hat{D}_e\psi_f+4(\Gamma^{cd}\epsilon_2)_\mu\bar{\epsilon_1}\Gamma_{[ba}\hat{D}_c\psi_{d]} \\
&+(\Gamma^k\epsilon_2)_\mu 2\bar{\epsilon_1}(-\Gamma_a\hat{D}_{[k}\psi_{b]}+\Gamma_k\hat{D}_{[b}\psi_{a]}+\Gamma_b\hat{D}_{[k}\psi_{a]})+\bar{\epsilon_2}\epsilon_1\frac{438}{7}(\hat{D}_{[b}\psi_{a]})_\mu \\
&+\bar{\epsilon_2}\Gamma^{ijk}\epsilon_1\{\frac{1}{28}(\Gamma_{bkj}\hat{D}_{[a}\psi_{i]}-(a\leftrightarrow b))_\mu-\frac{31}{42}(\Gamma_{kji}\hat{D}_{[b}\psi_{a]})_\mu\} \\
&+\bar{\epsilon_2}\Gamma^{ijkl}\epsilon_1\{-\frac{1}{6}(\Gamma_{balk}\hat{D}_j\psi_i)_\mu+\frac{1}{21}(\Gamma_{blkj}\hat{D}_{[a}\psi_{i]}-(a\leftrightarrow b))_\mu \\
&-\frac{1}{6}(\Gamma_{lkji}\hat{D}_{[b}\psi_{a]})_\mu-\frac{17}{84}(\delta_{la}\Gamma_{kj}\hat{D}_{[b}\psi_{i]}-(a\leftrightarrow b))_\mu \\
&+\frac{1}{3}(\delta_{la}\delta_{kb}-(a\leftrightarrow b))(\hat{D}_j\psi_i)_\mu\}. \qquad (3.3)
\end{aligned}
$$

Thus we obtain

$$
\begin{aligned}
\Lambda_{ab}{}^{(2)} &= \bar{\theta}\Gamma^n\epsilon\bar{\theta}\Gamma^k\psi_n\hat{\omega}_{kba}-\frac{1}{144}\bar{\theta}\Gamma^n\epsilon\bar{\theta}(\Gamma_{ba}{}^{cdef}\hat{F}_{cdef}+24\Gamma^{cd}\hat{F}_{bacd})\psi_n \\
&+\frac{1}{64}[\bar{\theta}\theta(-\frac{14016}{7}\bar{\epsilon}\hat{D}_{[b}\psi_{a]}) \\
&+\frac{1}{6}\bar{\theta}\Gamma^{xyz}\theta\{-\frac{48}{7}\bar{\epsilon}(\Gamma_{bzy}\hat{D}_{[a}\psi_{x]}-(a\leftrightarrow b))+\frac{992}{7}\bar{\epsilon}(\Gamma_{zyx}\hat{D}_{[b}\psi_{a]})\}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{24}\bar{\theta}\Gamma^{wxyz}\theta\{128\bar{\epsilon}\Gamma_{bazy}\hat{D}_x\psi_w - \frac{256}{7}\bar{\epsilon}(\Gamma_{bzyx}\hat{D}_{[a}\psi_{w]} - (a \leftrightarrow b)) \\
&+128\bar{\epsilon}\Gamma_{zyxw}\hat{D}_{[b}\psi_{a]} + \frac{1088}{7}\bar{\epsilon}(\delta_{za}\Gamma_{yx}\hat{D}_{[b}\psi_{w]} - (a \leftrightarrow b)) \\
&-256\bar{\epsilon}(\delta_{za}\delta_{yb}\hat{D}_x\psi_w - (a \leftrightarrow b))\}]. \qquad (3.4)
\end{aligned}
$$

${\Omega_{\mu ab}}^{(2)}$ is subject to the following equation,

$$
\begin{aligned}
\epsilon^\nu\partial_\nu{\Omega_{\mu ab}}^{(2)} &= \frac{1}{144}\bar{\theta}\Gamma^n\epsilon\bar{\psi}_n({\Gamma_{ba}}^{cdef}\hat{F}_{cdef} + 24\Gamma^{cd}\hat{F}_{bacd})_\mu - \bar{\theta}\Gamma^n\epsilon(\Gamma^k\psi_n)_\mu\hat{\omega}_{kba} \\
&-(\Gamma^n\epsilon)_\mu\bar{\theta}\Gamma^k\psi_n\hat{\omega}_{kba} - (\Gamma^n\epsilon)_\mu 2\bar{\theta}(\Gamma_b\hat{D}_{[a}\psi_{n]} - \Gamma_a\hat{D}_{[b}\psi_{n]} - \Gamma_n\hat{D}_{[b}\psi_{a]}) \\
&-\frac{1}{144}(\Gamma^n\epsilon)_\mu\bar{\theta}({\Gamma_{ab}}^{cdef}\hat{F}_{cdef} + 24\Gamma^{cd}\hat{F}_{abcd})\psi_n + \partial_\mu{\Lambda_{ab}}^{(2)}. \qquad (3.5)
\end{aligned}
$$

Thus we obtain

$$
\begin{aligned}
{\Omega_{\mu ab}}^{(2)} &= -\frac{1}{64}[\bar{\theta}\theta\frac{14212}{7}(\hat{D}_{[b}\psi_{a]})_\mu \\
&+\frac{1}{6}\bar{\theta}\Gamma^{xyz}\theta\{-\frac{120}{7}(\Gamma_{bzy}\hat{D}_{[a}\psi_{x]} - (a \leftrightarrow b))_\mu - \frac{1020}{7}(\Gamma_{zyx}\hat{D}_{[b}\psi_{a]})_\mu \\
&-192(\delta_{za}\Gamma_b\hat{D}_y\psi_x - (a \leftrightarrow b))_\mu - 48(\delta_{za}\Gamma_y\hat{D}_{[b}\psi_{x]} - (a \leftrightarrow b))_\mu\} \\
&+\frac{1}{24}\bar{\theta}\Gamma^{wxyz}\theta\{\frac{32}{7}(\Gamma_{bzyx}\hat{D}_{[a}\psi_{w]} - (a \leftrightarrow b))_\mu - 132(\Gamma_{zyxw}\hat{D}_{[b}\psi_{a]})_\mu \\
&-256(\delta_{za}\Gamma_{by}\hat{D}_x\psi_w - (a \leftrightarrow b))_\mu + \frac{32}{7}(\delta_{za}\Gamma_{yx}\hat{D}_{[a}\psi_{w]} - (a \leftrightarrow b))_\mu\}]. \qquad (3.6)
\end{aligned}
$$

4 Discussion

We have obtained ${\Lambda_{ab}}^{(2)}, {\Omega_{\mu a}}^{b(2)}$. Up to second order in anticommuting coordinates, only ${\Omega_{ma}}^{b(2)}$ remains. This component is complicated. However, it contains $\delta_{susy}\hat{D}_{[m}\psi_{n]}$ thus it is expected to contain curvature terms. From Bianchi identity, curvature terms should appeared in vielbein superfields at third and the higher order in anticommuting coordinates. This gives interaction terms coupled to curvature in Matrix model and higher curvature corrections to Einstein gravity in low energy effective actions. Thus it is important that we investigate ${\Omega_{ma}}^{b(2)}$. These terms and terms which are required to obtain terms of Matrix theory which are third order in anticommuting coordinates is under considerations.

However, ${\Omega_{\mu a}}^{b(2)}$ is also important in study of superspace structure and curved membrane action. κ-symmetry constraints act on torsion fields and curvature fields. Torsion

are defined as $T^A = DE^A = dE^A + E^B\Omega_B{}^A$. Thus $\Omega_{\mu a}{}^{b(2)}$ has information about more higher components of vielbein than $\Omega_{ma}{}^{b(2)}$. Moreover, $\Omega_{\mu a}{}^{b(2)}$ contains $\hat{D}_{[a}\psi_{b]}$ which is nonlinearly exact supersymmetry field strength which information is important in study of supersymmetry.

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Appendix

A Conventions

A.1 Indices

We use Greek indices for spinorial components and Latin indices for vector components. And we use former alphabet for the tangent space indices and later for general coordinates indices: $a, b, c, ...$ for tangent vector indices and $k, l, m, ...$ for general vector indices, and $\alpha, \beta, ...$ for tangent spinorial indices and $\mu, \nu, ...$ for general spinorial indices.

Superspace coordinates (x^m, θ^μ) are designated Z^M , where later capital Latin alphabet $M, N, ..$ are collective designations for general coordinate indices. While former capital Latin alphabet $A, B, ..$ are collective designations for tangent space indices.

A.2 p-form superfield

We introduce p-form superfields as follows,

$$
\begin{aligned}
X &\equiv \frac{1}{p!} dz^{M_p} ... dz^{M_1} X_{M_p ... M_1} \\
&\equiv \frac{1}{p!} E^{A_p} ... E^{A_1} X_{A_p ... A_1}, \qquad \text{(A.1)} \\
X_{A_p ... A_1} &\equiv \sum_{i=1}^{32} X_{A_p ... A_1}{}^{(i)}. \qquad \text{(A.2)}
\end{aligned}
$$

$X_{A_p ... A_1}{}^{(i)}$ is component at i-th order in anticommuting coordinates.

A.3 Brackets

Symmetrization bracket ( ) and antisymmetrization bracket [ ] is defined as follows,

$$
\begin{aligned}
[M_1...M_N] &= \frac{1}{N!}(M_1...M_N \quad + \text{antisymmetric terms}), \\
(M_1...M_N) &= \frac{1}{N!}(M_1...M_N \quad + \text{symmetric terms}).
\end{aligned} \tag{A.3}
$$

A.4 Gamma matrices(11-dimensional)

Since we use the Majorana representation, all components are real.

Gamma matrix $\Gamma^{a\ \alpha}{}_{\beta}$ is defined as follows,

$$
\{\Gamma^a, \Gamma^b\} = 2\eta^{ab}. \tag{A.4}
$$

We use the mostly plus metric; $\eta_{ab} \sim (-+...+)$. We lower the spinorial indices by charge conjugation matrix $C_{\alpha\beta}$.

$$
\begin{aligned}
\bar{\psi}_\beta &= \psi^\alpha C_{\alpha\beta}, \\
\Gamma^a{}_{\alpha\beta} &= C_{\alpha\gamma}\Gamma^{a\ \gamma}{}_{\beta}.
\end{aligned} \tag{A.5}
$$

$\Gamma^{a_1..a_n}{}_{\alpha\beta}(n = 1, 2, 5, 6, 9, 10)$ are symmetric matrices and $\Gamma^{a_1..a_n}{}_{\alpha\beta}(n = 0, 3, 4, 7, 8, 11)$ are antisymmetric matrices.

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